Section 6.3
Antidifferentiation by Parts

Product Rule for Differentiation

$$\int \frac{d}{dx}(uv)\,dx = \int u\frac{dv}{dx}\,dx + \int v\frac{du}{dx}\,dx$$

Product Rule in Integral Form

$$\int \left(u\frac{dv}{dx}\right)dx = \int \left(\frac{d}{dx}uv\right)dx - \int \left(v\frac{du}{dx}\right)dx$$

uv

Integration by Parts Formula

$$\int u\,dv = uv - \int v\,du$$

Example

Evaluate $\int x \cos x \, dx =$

u dv

$u = x$ $\qquad dv = \cos x \, dx$

$\frac{du}{dx} = 1$ $\qquad v = \int \cos x \, dx$

$du = dx$ $\qquad v = \sin x$

$$\int x\, e^x\, dx =$$

$u = x \qquad dv = e^x\, dx$

$du = dx \qquad v = \int e^x\, dx = e^x$

$$\int x e^x dx = x e^x - \int e^x\, dx$$

$$= x e^x - e^x + C$$

$$\int x^2 \ln x \, dx =$$

$u = \ln x \qquad dv = x^2 dx$

$du = \frac{1}{x} dx \qquad v = \int x^2 dx = \frac{x^3}{3}$

$$\int x^2 \ln x \, dx = \frac{x^3}{3} \ln x - \frac{1}{3} \int x^3 \cdot \frac{1}{x} dx$$

$$= \frac{x^3}{3} \ln x - \frac{1}{3} \int x^2 dx$$

$$= \frac{x^3}{3} \ln x - \frac{x^3}{9} + C$$

$$\int \ln x \, dx =$$

$$\int x^2 e^x \, dx =$$

The Order

L I P E T

LOG INV TRIG POLY EXP TRIG

Solve the differential equation $dy/dx = x \ln(x)$ subject to the initial condition $y = -1$ when $x = 1$. Confirm the solution graphically by showing that it conforms to the slope of the field.

Evaluate

$$\int e^x \cos x \, dx$$

$$\int 2x\, e^{3x}\, dx$$

$$\int x^2 \cos 3x \, dx$$

$$\int e^{2x} \sin x \, dx =$$

$$\int x \csc^2 x \, dx$$

$$\int e^{3x} \cos x \, dx =$$

$$\int e^x \cos 2x \, dx =$$

Tabular Integration

- Shortcut for Integration by Parts

Only if u is a polynomial

Evaluate $\int x^2 e^x \, dx$

With $f(x) = x^2$ and $g(x) = e^x$, we list:

$f(x)$ and its derivatives		$g(x)$ and its integrals
x^2	$(+)$ →	e^x
$2x$	$(-)$ →	e^x
2	$(+)$ →	e^x
0	→	e^x

$$\int x^4 e^{3x}\, dx =$$

$$\int x^4 e^{(1/2)x}\, dx$$

$$\int x^3 \sin x \, dx$$

$$\int_{0}^{2} (x^2 - 5x)\, e^x \, dx =$$

$$\int \frac{x^2 - 9}{x} \, dx$$

$$\int \frac{x}{x^2 - 9}\, dx$$